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An Evaluation of Theories for Predicting Turbulent Skin Friction and Heat Transfer on Flat Plates at Supersonic and Hypersonic Mach Numbers

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Nomenclature

C_f = local skin-friction coefficient, τ_w/q_e
C_F = average skin-friction coefficient, $2\theta/x$
C_h = local Stanton number, $\dot{q}_w/\rho_e U_e(H_w - H_{aw})$
C_p = specific heat at constant pressure
F = T_w/T_e
F_{aw} = T_w/T_{aw}
F_c = transformation function, $\bar{C}_f/C_f$
F_θ = transformation function, $\bar{R}e_\theta/Re_\theta$
F_x = transformation function, $\bar{R}e_x/Re_x$
H = total enthalpy
H_{aw} = adiabatic wall enthalpy; see Eq. (44)
m = $0.2M_e^2$
M = Mach number
Pr = Prandtl number
Pr_T = turbulent Prandtl number
q = dynamic pressure
$\dot{q}_w$ = rate of heat transfer from the surface per unit area
r = recovery factor, 0.9
R = local-cone radius or wind-tunnel radius
Re_x = Reynolds number based on distance to virtual origin of turbulent flow, $\rho_e U_e x/\mu_e$
Re_Γ = Reynolds number based on energy thickness, $\rho_e U_e \Gamma/\mu_e$
Re_θ = Reynolds number based on momentum thickness, $\rho_e U_e \theta/\mu_e$
s = distance along surface from cone apex or flat-plate leading edge
t = $1 - F_{aw}$
T = absolute temperature
U = velocity
U_τ = shearing velocity, $(\tau_w/\rho_w)^{1/2}$
x = distance along surface from virtual origin of turbulent flow
x_L = distance along surface from leading edge to peak Stanton number location
x_T = distance along surface from virtual origin of turbulent flow to peak Stanton number location
y = distance normal to surface
$\bar{z}$ = transformed normal coordinate, $\bar{y}\bar{U}_\tau/\bar{\nu}$
Γ = energy thickness, $\int_0^\delta \frac{\rho U}{\rho_e U_e}\frac{H - H_e}{H_w - H_e}dy$; also see Eq. (41)
δ = boundary-layer thickness
ϵ_H = eddy thermal conductivity
ϵ_V = eddy viscosity
ϵ = index, $\epsilon = 0$ for flat plate and $\epsilon = 1$ for cone

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θ = momentum thickness, $\int_0^\delta \frac{\rho U}{\rho_e U_e} 1 - \frac{U}{U_e} dy$ for flat plates or $\int_0^\delta \frac{\rho U}{\rho_e U_e} 1 - \frac{U}{U_e} \frac{R-y}{R} dy$ for circular wind-tunnel walls
μ = coefficient of viscosity
ν = coefficient of kinematic viscosity
ρ = mass density
τ = shear stress

Subscripts

aw = adiabatic wall
C = Coles
e = boundary-layer edge
exp = experimental
i = incompressible
L = laminar
max = maximum
SC = Spalding and Chi
SS = Sommer and Short
t = total
T = turbulent
the = theoretical
VD = Van Driest
w = wall

Superscript

$(\bar{\ })$ = variable transformed to equivalent constant flow property case

Introduction

ACCURATE knowledge of skin friction and heat transfer is required to predict the performance and structural requirements of supersonic and hypersonic aircraft. Generally, theory is relied upon to provide estimates of either wind-tunnel or flight values of local skin friction since it is impractical to measure these values over the entire configuration. Similarly, thermal design of airplane structures and material selection depend largely on predictions of incoming convective heat flux over the various surfaces. At supersonic speeds ($M \simeq 3$), the surface temperature is essentially the adiabatic wall temperature. At hypersonic speeds ($M \simeq 7$), however, the external surface temperatures generally will be 0.3–0.5 of the adiabatic wall temperatures as a result of considerable radiative cooling and internal heat transfer. Since all turbulent skin-friction and heat-transfer theories are dependent upon experimental data for determining certain constants, it follows that any theory evaluation will depend on the accuracy of the data and the method by which the data are reduced. Previous evaluations included skin-friction data reduced by indirect methods for lack of data directly measured by balances, particularly at hypersonic Mach numbers on nonadiabatic walls. Indirect methods include those in which skin friction is derived from 1) heat-transfer measurements with an assumed Reynolds analogy factor[1,2]; 2) the rate of change of momentum thickness with longitudinal distance[3–8]; 3) the velocity gradient at the surface[9–13]; and 4) the velocity profile.[14–17] Examples of evaluations using both direct and indirect measurements are those of Spalding and Chi,[18,19] Peterson,[20] and Hopkins and Keener.[21] Spalding and Chi examined 20 existing theories* on the basis of the root-mean-square (rms) difference between measured and predicted values of local skin-friction coefficient, and found that the theories of Sommer and Short,[22] Kutateladze and Leont'ev,[23] Wilson,[24] and Van Driest II[25]† gave the best results. However, Spalding and Chi developed a semiempirical theory which gave an rms difference slightly less than all the aforementioned theories. Since the Kutateladze and Leont'ev theory is considerably more complex than the other theories, it was not recommended.[19] Miles and Kim[26] indicated that Coles' theory,[14] not included in the Spalding and Chi analysis, is competitive with Spalding and Chi's theory on an rms difference basis. For the adiabatic-wall case, analyses of Hopkins and Keener[21] and Peterson[20] indicated that the theories of Van Driest II (Ref. 25) and Sommer and Short[22] generally bracketed all the experimental results, but that near Mach number 6 these theories differ by as much as 25%.

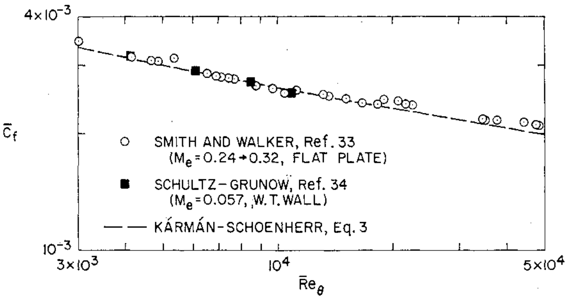

Fig. 1 Incompressible adiabatic skin friction measured on a flat plate and a wind-tunnel wall; C_f and Re_θ directly measured.

The theories of Sommer and Short,[22] Spalding and Chi,[19] Van Driest II,[25] and Coles[14] were reevaluated recently by Hopkins et al.[27] on the basis of skin friction which was directly measured by balances mounted on both adiabatic and cold-wall surfaces. Results from this study which covered both supersonic and hypersonic cases indicate that for $T_w/T_{aw} > 0.3$, theories of Van Driest II or Coles predicted the measured skin friction generally within about $\pm 10\%$. Theories of Sommer and Short and Spalding and Chi generally underpredicted the measured skin friction at high Mach numbers by 20 to 30%. For $T_w/T_{aw} < 0.3$, none of the theories gave good predictions of the skin friction.

Heat-transfer data for flat plates are compared by Bertram et al.[1,2] with theoretical values derived from skin-friction theory through an extension to compressible flow of the von Kármán form of the Reynolds analogy factor. Bertram's results and later results of Cary[28] suggest best agreement with the prediction method of Spalding and Chi. A similar result is reported by Hopkins et al.[27] provided a Reynolds analogy factor of 1.16, close to that used by Bertram and Cary, is assumed. However, if a Reynolds analogy factor of 1.0 as measured by Polek and Keener[27] is used, then the heat-transfer results favor the theories of Van Driest II or Coles, a result consistent with the skin-friction-theory evaluation.

This survey article is based on the presentation of Ref. 27‡ but is extended to include the skin friction presented on a generalization basis [i.e., transformed to the incompressible plane or $\bar{C}_f = f(\bar{R}e_\theta)$] and some additional skin-friction and heat-transfer data. Directly measured C_f from balances and Re_θ (when available) were chosen for evaluating the turbulent skin-friction theories of Sommer and Short,[22] Spalding and Chi,[19] Van Driest II,[25] and Coles.[14] It was decided to make the evaluations on the basis of the momentum thickness Reynolds number (Re_θ) as proposed by several authors, e.g., Squire and Young[29] and Coles.[30] This approach assumes that the boundary layer is fully turbulent with a unique relationship between the skin-friction coefficient (C_f) and Re_x or Re_θ for given values of M_e, $T_{t,e}$, and T_w/T_{aw}. Use of Re_θ rather than Re_x avoids having to determine arbitrarily the origin of

* Since empiricism is involved in the evaluation of constants for any turbulent skin-friction theory, no distinction will be made herein between a so-called theory and method.

† The "II" refers to the second theory of Van Driest in which the von Kármán mixing length is used.

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turbulent flow. In an analogous manner, Re_Γ was chosen for analyzing the heat-transfer results for the same theories. The present evaluation is restricted to flat plates, cones, and wind-tunnel walls on which the flow is essentially isothermal and isobaric. Since all the above theories essentially involve a transformation of C_f and Re_θ to the incompressible plane, the selection of the proper incompressible skin-friction formula is discussed.

Incompressible Skin-Friction Formula

For the evaluation of the skin-friction theories that are discussed later, it was necessary to choose a single incompressible skin-friction formula to be consistent.

From mixing-length considerations, von Kármán[31] pointed out that at high Reynolds numbers, C_f can be expressed by an equation in which $C_f = f[\log(Re_xC_f)]$. A similar equation with an experimentally derived constant was developed for average skin friction by Schoenherr[32] and is given below:

$$0.242/\bar{C}_F^{1/2} = \log_{10}(\bar{R}e_x\bar{C}_F) = \log_{10}(2\bar{R}e_\theta) \tag{1}$$

Schoenherr also derived a relationship between the local and average skin friction as

$$\bar{C}_f = 0.242\bar{C}_F/(0.242 + 0.8686\bar{C}_F^{1/2}) \tag{2}$$

From Eqs. (1) and (2) the following equation can be written relating $\bar{C}_f$ to $\bar{R}e_\theta$:

$$(1/\bar{C}_f) = 17.08(\log_{10}\bar{R}e_\theta)^2 + 25.11 \log_{10}\bar{R}e_\theta + 6.012 \tag{3}$$

Equation (3), the Kármán-Schoenherr equation in terms of Re_θ, was chosen for the evaluation of all skin-friction theories considered herein. Although this equation is shown in Fig. 1 to give a good representation of the direct skin-friction measurements of Smith and Walker[33] and Schultz-Grunow,[34] additional direct measurements appear warranted at higher and lower Reynolds numbers. The Kármán-Schoenherr Eq. (1) for average skin friction was verified by Locke[35] over the range $4 \times 10^5 < \bar{R}e_x < 5 \times 10^8$ and is recommended by Locke to be universally adopted. The only reservation that Locke had regarding Eq. (1) is that the coefficients were derived from test results not corrected for three-dimensional edge effects. This correction is not expected to be greater than 5%. Results from several other formulas for computing the local skin friction are compared with results from the Kármán-Schoenherr formula in Table 1. Formulas used for $\bar{C}_f$ in Table 1 are given below:

Blasius[36]:

$$\bar{C}_f = 0.026/Re_\theta^{1/4} \tag{4}$$

Clauser[37]:

$$(2/\bar{C}_f)^{1/2} = 5.6 \log_{10}[\bar{R}e_\theta/\{1 - 6.8(\bar{C}_f/2)^{1/2}\}] + 4.226 \tag{5}$$

Sivells & Payne[38]:

$$\bar{C}_f = 0.088(\log_{10}\bar{R}e_x - 2.3686)/(\log_{10}\bar{R}e_x - 1.5)^3 \tag{6}$$

$$\bar{R}e_\theta = 0.044\bar{R}e_x/(\log_{10}\bar{R}e_x - 1.5)^2 \tag{7}$$

Spalding & Chi[19]:

$$\bar{R}e_\theta = (u^+)^2/6 + [(1 - 2/ku^+)e^{ku^+} + 2/ku^+ + 1.0 - (ku^+)^2/6 - (ku^+)^3/12 - (ku)^+)^4/40 - (ku^+)^5/180]/kE \tag{8}$$

where $k = 0.4$, $E = 12$, $u^+ = (2/\bar{C}_f)^{1/2}$. Both the Sivells-Payne and Spalding-Chi formulas show good agreement with the Kármán-Schoenherr formula at all Reynolds numbers, but the Blasius and Clauser formulas show unfavorable agreement at some Reynolds numbers.

Table 1 Percent difference in $\bar{C}_f$ from the Kármán-Schoenherr values given by Eq. (3)

$\bar{R}e_\theta$	Blasius	Clauser	Sivells & Payne	Spalding & Chi
10^2	2.4	−10.7	0.3	−2.0
4×10^2	8.7	−4.8	−2.0	−3.1
10^3	8.6	−2.3	−2.1	−2.0
4×10^3	4.0	0.4	−1.5	−0.4
10^4	−1.3	1.7	−0.8	0.5
4×10^4	−11.2	3.1	0.5	1.4
10^5	−18.4	3.8	1.3	1.8

Compressible Skin-Friction Theories

C_f and Re_θ Transformations

The theories will be evaluated on a generalized basis by determining how well each theory transforms the measured skin-friction coefficients onto the Kármán-Schoenherr $\bar{C}_f$-$\bar{R}e_\theta$ curve. Such a comparison permits experimental points obtained at different test conditions to be examined together. In this procedure the transformed and measured skin-friction coefficients and momentum-thickness Reynolds numbers are related as follows:

$$\bar{C}_f = F_cC_f \tag{9}$$

$$\bar{R}e_\theta = F_\theta Re_\theta \tag{10}$$

where the transformation functions F_c and F_θ for each theory are presented later in this section.

For those experiments in which Re_θ was not measured, it was necessary to derive Re_θ from Re_x. This was accomplished by iteration in the equation formed by equating the Kármán-Schoenherr equations, Eqs. (2) and (3), after substituting $2\bar{R}e_\theta/\bar{R}e_x$ for $\bar{C}_F$ in the former. Again, the Kármán-Schoenherr equation was chosen for evaluating all theories for consistency. It can be shown from the momentum integral equations for the compressible and transformed flows, respectively,

$$C_f/2 = dRe_\theta/dRe_x \tag{11}$$

$$\bar{C}_f/2 = d\bar{R}e_\theta/d\bar{R}e_x \tag{12}$$

so that

$$\bar{R}e_x = \int_0^{Re_x} \left(\frac{F_\theta}{F_c}\right) dRe_x \tag{13}$$

For the theories of Sommer and Short, Spalding and Chi, and Van Driest, the functions F_θ and F_c do not vary with x; hence a transformation function F_x can be defined as

$$F_x = \bar{R}e_x/Re_x = F_\theta/F_c \tag{14}$$

For Coles' theory, however, F_θ and F_c vary with x so that this simplification is not possible, and an integration must be performed to determine $\bar{R}e_x$.

Sommer and Short[22]

It is assumed that C_f and Re_θ can be transformed to incompressible values provided the density and viscosity are evaluated at some suitable reference temperature which is a function of M_e, T_w/T_e, and T_e. This temperature, derived from limited experimental drag data obtained in free flight, is given by

$$T_{SS}' = T_e[1 + 0.035M_e^2 + 0.45(F - 1)] \tag{15}$$

and the transformation functions

$$(F_c)_{SS} = T'_{SS}/T_e \tag{16}$$

$$(F_\theta)_{SS} = \mu_e/\mu_{SS}' \tag{17}$$

$$(F_x)_{SS} = \mu_eT_e/\mu_{SS}'T_{SS}' \tag{18}$$

in which the viscosity for all theories was calculated by Keyes' formula

$$\mu = 0.0232 \times 10^{-6} \times T^{1/2}/[1 + (220/T) \times 10^{-9/T}] \text{ lb-sec/ft}^2 \quad (19)$$

in which T is in °R.

Van Driest II[25]

The von Kármán mixing length is used in the Prandtl shear stress equation. The well-known Crocco temperature distribution through the boundary layer is assumed. For the calculations herein, a temperature recovery factor of $r = 0.9$ was used in the transformation functions for C_f and Re_x in a manner given by Spalding and Chi.[19] The transformation functions are

$$(F_c)_{\mathrm{VD}} = rm/(\sin^{-1}\alpha + \sin^{-1}\beta)^2 \quad (20)$$

$$(F_\theta)_{\mathrm{VD}} = \mu_e/\mu_w \quad (21)$$

$$(F_x)_{\mathrm{VD}} = (\mu_e/\mu_w)(\sin^{-1}\alpha + \sin^{-1}\beta)^2/rm \quad (22)$$

where

$$\alpha = (2A^2 - B)/(4A^2 + B^2)^{1/2} \quad (23)$$

$$\beta = B/(4A^2 + B^2)^{1/2} \quad (24)$$

$$A = (rm/F)^{1/2} \quad (25)$$

$$B = (1 + rm - F)/F \quad (26)$$

Spalding and Chi[19]

The function for transforming C_f is the same as derived by Van Driest II provided the same temperature recovery factor is included [see Eq. (20)]. The transformation functions for Re_θ and Re_x were assumed to be made up of the factors $(T_w/T_{aw})^a$ and $(T_w/T_e)^b$ where a and b are exponents determined empirically. Unfortunately, a and b were derived primarily in Ref. 19 from indirect measurements of skin friction for the heat-transfer case for lack of direct measurements. For this theory, the transformation functions are

$$(F_\theta)_{\mathrm{SC}} = 1/(F^{0.702}F_{aw}{}^{0.772}) \quad (27)$$

$$(F_x)_{\mathrm{SC}} = (\sin^{-1}\alpha + \sin^{-1}\beta)^2/(rmF^{0.702}F_{aw}{}^{0.772}) \quad (28)$$

Coles[14]

A boundary-layer substructure hypothesis is invoked in which a substructure Reynolds number based on a suitable mean temperature is adopted as constant. This mean temperature was derived by Coles from an assumed Dorodnitsyn-Howarth density scaling for the normal coordinate and several direct measurements of skin friction at $0.2 < M_e < 5.8$. The resulting transformation functions are

$$(F_c)_C = T_w\mu_C/T_e\mu_w \quad (29)$$

$$(F_\theta)_C = \mu_e/\mu_C \quad (30)$$

$$(F_x)_C = \frac{1}{Re_x}\int_0^{Re_x} (T_e\mu_e\mu_w/T_w\mu_C{}^2)dRe_x \quad (31)$$

where

$$T_C = \left(\frac{T_e}{430}\right)\int_0^{430}\left(\frac{T}{T_e}\right)_C d\bar{z} \quad (32)$$

and

$$(T/T_e)_C = F + (1 + m - F)(\bar{U}/\bar{U}_\tau)[(\bar{C}_f/2)_C]^{1/2} - (\bar{U}/\bar{U}_\tau)^2 m(\bar{C}_f/2)_C \quad (33)$$

$\bar{U}/\bar{U}_\tau$ is given as a function of $\bar{z}$ in Ref. 39; T_C is obtained by iteration from Eqs. (3, 30, and 32).

An alternate choice to the substructure hypothesis is the sublayer hypothesis due to Donaldson[40] which was employed by Baronti and Libby.[16] They patterned their application of the sublayer hypothesis after Coles except that the Reynolds number associated with the laminar sublayer was taken as invariant. It is shown[41] that the Baronti-Libby transformation gave an underprediction of skin friction of about 20% at a Mach number of 6.5; therefore, this theory is not included herein.

C_f Predictions

Theoretical values of C_f can be calculated from the transformations given previously for each theory and the Kármán-Schoenherr equation, Eq. (3). This procedure will be illustrated for the Sommer and Short theory. First, the reference temperature (T'_{SS}) is calculated from Eq. (15). Then, for the experimental value of Re_θ, the transformed $(Re_\theta)_{\mathrm{SS}} = (F_\theta)_{\mathrm{SS}}Re_\theta$ is calculated from Eq. (17). Finally, the theoretical C_f is calculated from the following equation derived from Eqs. (3) and (16):

$$(C_{f,\mathrm{the}})_{\mathrm{SS}} = (T_e/T'_{\mathrm{SS}})/\{17.08[\log_{10}(\bar{R}e_\theta)_{\mathrm{SS}}]^2 + 25.11\log_{10}(\bar{R}e_\theta)_{\mathrm{SS}} + 6.012\} \quad (34)$$

Heat-Transfer Theories

The approach adopted herein for predicting heat transfer is to use a Reynolds analogy factor in conjunction with one of the previously discussed skin-friction theories. A dilemma is that the skin-friction theories differ appreciably for certain conditions, particularly those applicable to a hypersonic aircraft. Consequently, various combinations of Reynolds analogy factors and skin-friction theories can be used to predict either the same or different heat-transfer rates. Any comparison of heat-transfer data with theory is therefore contingent on establishing the proper Reynolds analogy factor.

Reynolds Analogy Factor

A complete theory for variable property turbulent boundary layers would predict the heat transfer as well as the skin friction. Such an analysis requires some hypothesis for the energy transferred by the turbulent eddies, analogous to the Reynolds stress terms in the momentum equation. For example, a turbulent Prandtl number may be defined as

$$Pr_T = C_p\epsilon_V/\epsilon_H \quad (35)$$

where ϵ_V and ϵ_H are defined as the eddy viscosity and eddy thermal conductivity, respectively. Appropriate assumptions for Pr_T lead to solutions of the energy equation in terms of the velocity or shear-stress distributions. The results may then be expressed in terms of a Reynolds analogy factor relating the Stanton number to the skin-friction coefficient, viz.,

$$\text{Reynolds analogy factor} = 2C_h/C_f \quad (36)$$

For $Pr = Pr_T = 1$, a solution to the energy equation is the Crocco relationship

$$(H - H_w)/(H_e - H_w) = U/U_e \quad (37)$$

The Reynolds analogy factor that results is

$$2C_h/C_f = 1 \quad (38)$$

which was first proposed in 1874 by Reynolds.[42] Subsequent theories incorporating various assumptions for the boundary-layer structure and Pr_T are summarized by Rubesin.[43]

At hypersonic speeds and with wall temperatures somewhat less than the adiabatic value, a current procedure is to assume the same Reynolds analogy factor as is used at the lower speeds. For example, Bertram and Neal[1] suggest the von Kármán formula[44] with C_f replaced by the transformed value,

$F_c C_f$. Empirical recommendations for Reynolds analogy factor are presented under Results and Discussion.

Energy Thickness

The heat-transfer data are analyzed in a manner analogous to the skin-friction data. The thermal boundary layer is assumed to be fully turbulent; hence the local Stanton number can be expressed as a unique function of an energy thickness Reynolds number for given values of M_e, $T_{t,e}$, and T_w/T_{aw}. The energy thickness is defined as

$$\Gamma \equiv \int_0^\delta \frac{\rho U}{\rho_e U_e} \frac{H - H_e}{H_w - H_e} \, dy \tag{39}$$

and is related to the local Stanton number through the energy integral equation

$$C_h(H_w - H_{aw})/(H_w - H_e) \equiv \dot{q}_w/\rho_e U_e(H_w - H_e) = d\Gamma/ds + \Gamma(d/ds)\{\log_e[R^\epsilon \rho_e U_e(H_w - H_e)]\} \tag{40}$$

Instead of boundary-layer surveys as for the momentum thickness, the measured surface heat-transfer rates can be used to evaluate the energy thickness from

$$\Gamma = \int_0^s \dot{q}_w(s) R^\epsilon(s) ds/\rho_e U_e(H_w - H_e) R^\epsilon \tag{41}$$

which results from integration of Eq. (40). [For the Ames tests[27] where boundary-layer surveys were also obtained, the energy thicknesses from Eqs. (39)and (41) agreed within 7%.]

Heat-Transfer Predictions from Re_Γ

Prediction of turbulent boundary-layer heat transfer from skin-friction theories, $C_f(Re_\theta)$, depends on the relationship between Re_θ and Re_Γ through an assumed Reynolds analogy factor and temperature recovery factor. Choices of these factors are based either on extensions of the now classic constant-property theories or on experimental results. If these factors are constant along the surface, then Re_θ, Re_Γ, C_h, and C_f are related as follows:

$$Re_\theta = Re_\Gamma/[(2C_h/C_f)(H_w - H_{aw})/(H_w - H_e)] \tag{42}$$

$$(C_h)_{\text{the}} = (2C_h/C_f)(C_f/2)_{\text{the}} \tag{43}$$

where $2C_h/C_f$ is the Reynolds analogy factor, and r, the recovery factor, is introduced into the adiabatic wall enthalpy,

$$H_{aw} = H_e - (1 - r)U_e^2/2 \tag{44}$$

In Eq. (43), $(C_f/2)_{\text{the}}$ is calculated by a given theory for the Re_θ from Eq. (42) in which Re_Γ is $(\rho_e U_e/\mu_e)$ times Γ from Eq. (41).

Table 2 Skin friction for adiabatic flat plates[a] (Figs. 2 and 3)

Symbol	M_e	$Re_\theta \times 10^{-3}$	Ref.
⊓	1.5	2	49
△	1.63	12	50
▽	1.68	20	50
▷	1.73	13	50
◇	1.75	2	49
◹	1.97	6	51
◁	2.00	13	50
◺	2.11	14	50
◸	2.25	7	50
◿	2.46	12	50
◆	2.56	6	51
◇	2.80	81	52
◇	3.69	5	51
⌂	4.53	5	51
◊	5.79	3	53

[a]Boundary-layer trips used in all tests.

Table 3 Skin friction for adiabatic wind-tunnel walls (Figs. 4 and 5)

Symbol	M_e	$Re_\theta \times 10^{-3}$	Ref.
○	1.75	8	54
□	2.01	8	54
□	2.23	7	54
○	2.46	68	21
□	2.49	7	54
◇	2.67	690	52
□	2.72	7	54
◇	2.80	360	52
□	2.95	7	54
◑	2.95	15	55
○	2.96	61	21
□	3.16	7	54
□	3.39	7	54
○	3.45	56	21
□	3.67	6	54
◐	4.20	13	55
×	4.75	28	56

Results and Discussion

Skin Friction

Skin-friction results will be presented separately for experiments conducted on wind-tunnel walls and flat plates, since a significant difference in the boundary-layer temperature profiles has been shown to exist for these cases.[45,46] Seiff and Short[47] noticed a similar difference in temperature distributions on free-flight models and a nozzle wall and suggested that the rapid expansion in the nozzle to high Mach numbers results in a dynamically nonequilibrium boundary layer. In addition, adiabatic and nonadiabatic wall results are examined separately to isolate any special effects from heat transfer.

Only skin friction directly measured by balances was chosen for the evaluation with the exception of the Sommer and Short data[22] obtained from drag measurements of a circular cylinder fired down a ballistic range. For the latter data the local skin friction was calculated from the average skin friction by assuming that $C_f/C_{f,i} = C_F/C_{F,i}$ for the same Reynolds number. All the adiabatic data chosen included measurements of the momentum thickness Reynolds number and were analyzed on this basis. Such an analysis precludes the necessity of having to assume a virtual origin of turbulent flow. Because of the scarcity of nonadiabatic wall data, some data were selected for which the momentum thickness was not measured. The virtual origin of turbulent flow was derived by two currently used methods. In the first method, the virtual origin was assumed to be coincident with the location of the maximum Stanton number. This same assumption was made in Refs. 1, 2, and 28. In the second method, the virtual origin was derived from theory by assuming that the momentum thicknesses for turbulent and laminar flow at the end of transition are equal, i.e., $(x_T)(C_{F,T}) = (x_L)(C_{F,L})$, for which x_L was taken as the point where the Stanton number is maximum. This method is described in detail in Ref. 48.

Skin-friction data chosen for the evaluation are identified by symbols, flow conditions, and references in Tables 2–5. Although there are probably differences in the accuracy of the direct skin-friction measurements and the recording instrumentation for each experiment, the expected accuracy is generally within ±5%. The Reynolds numbers shown represent approximate average values for the data referenced. The results will first be presented on a generalized basis and then on a $(C_{f,\text{exp}}/C_{f,\text{the}} - 1)$ basis.

Adiabatic flat plates

Figure 2 indicates that for $1.1 \times 10^3 < Re_\theta < 1.5 \times 10^4$ all four theories generally transform the measured skin friction onto the Kármán-Schoenherr incompressible curve. At

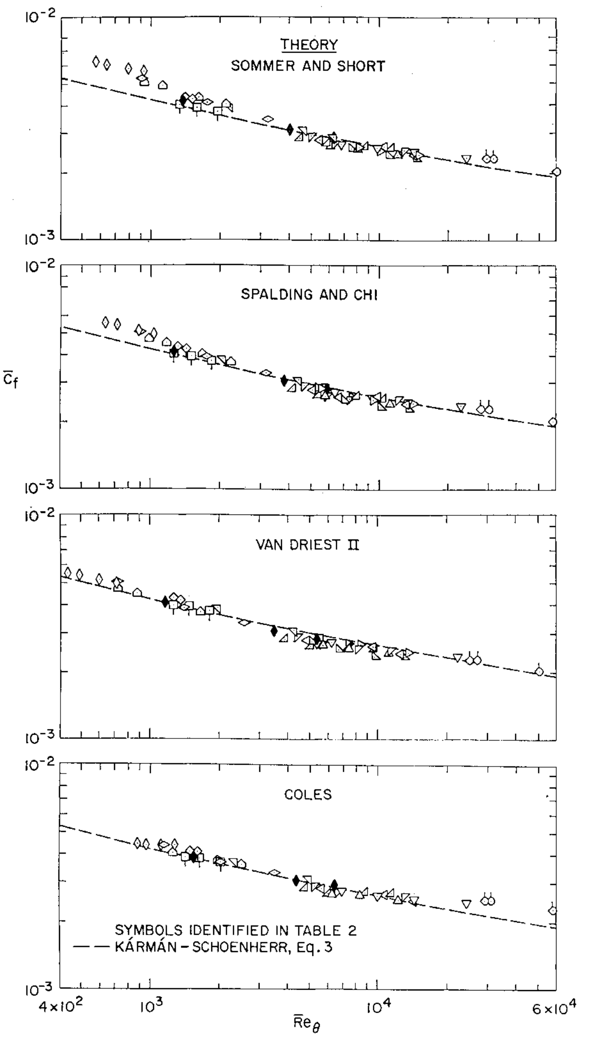

Fig. 2 Generalization of adiabatic-wall skin friction measured on flat plates; C_f and Re_θ directly measured; $M_e = 1.5 \to 5.8$.

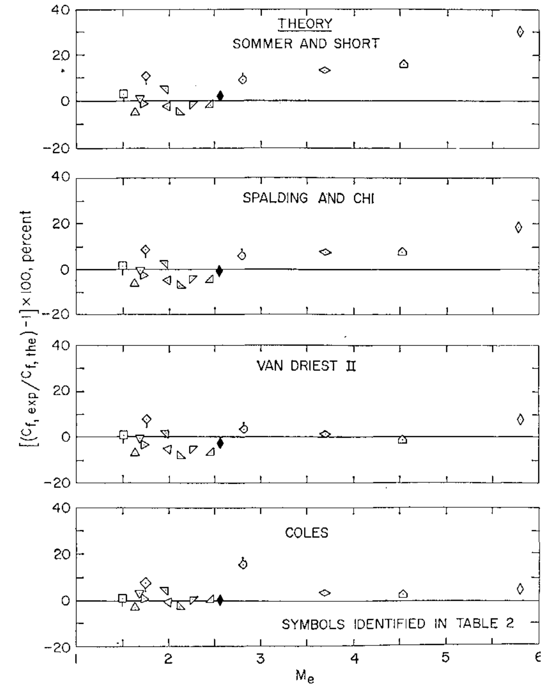

Fig. 3 Effect of Mach number on predictions of adiabatic-wall skin friction measured on flat plates; C_f and Re_θ directly measured.

higher values of Re_θ, however, the Mach 2.8 data of Moore and Harkness[52] (symbols with flags up) are transformed considerably above the incompressible curve by Coles' theory; therefore, these skin-friction data are underpredicted by about 15% as shown in Fig. 3. Since this high Reynolds number point falls out of line with the other points shown in Fig. 2, it appears that Coles' theory inherently underpredicts the skin friction at high Reynolds numbers because of the dependency of the substructure mean temperature on the Reynolds number [see Eqs. (32) and (33)]. At low values of Re_θ, the Mach 5.8 data of Korkegi[53] (diamond symbols) are considerably underpredicted by the theories of Sommer and Short[22] and Spalding and Chi[19] as shown in Fig. 2. This result is believed associated with the inadequacy of these theories at high Mach numbers rather than with the low Reynolds number since the theories of Van Driest II and Coles transform these data satisfactorily (see Fig. 3).

Table 4 Skin friction for nonadiabatic flat plates (Figs. 6 and 7)

Symbol	M_e	B. L. Trips	$Re_\theta\times10^{-3}$	$Re_x\times10^{-6}$	T_w/T_{aw}	Ref.
▽	2.8	Yes		2	0.43	22
▷	3.8	Yes		2	0.29	22
⌂	4.9	No	6		0.60 → 0.99	57
◇	5.6	Yes		3	0.20	22
○	6.5	Yes	5		0.32 → 0.51	41
●	6.5	No	4		0.34 → 0.51	41
✦	6.6	No	4		0.31	27
□	6.8	No		3	0.57	58[a]
⬚	6.8	No		2	0.57	58[a]
△	7.0	Yes		1	0.18	22
◺	7.4	No		14	0.14	48
◺	7.4	No		12	0.14	48
✱	7.4	No	4		0.31	27
⌂	4.53	Yes	5		1.0	51
◊	5.79	Yes	3		1.0	53

[a] □ Virtual origin derived from $(x_T)(C_{F,T}) = (x_L)(C_{F,L})$.
⬚ Virtual origin assumed coincident with the maximum Stanton number.

Adiabatic wind-tunnel walls

Although as pointed out before there can be a large difference in temperature distributions within the boundary layer on flat plates and wind-tunnel walls, it is shown by Adcock et al.[59] that for the adiabatic wall case the difference in Re_θ for linear and quadratic total temperature distributions is small. In viewing the results for wind-tunnel walls, it should be emphasized that the theories of Van Driest and Coles are based on the Crocco linear distribution of total temperature with velocity.

In general, as shown in Figs. 4 and 5, up to a Mach number of about 3.5 the theoretical transformations and skin-friction predictions for the wind-tunnel walls and flat plates were similar, i.e., all the theories gave predictions of skin friction to within about 10% of the measured values, except for Coles' theory at high Reynolds numbers. At the highest Mach number shown, the data of Lee et al.[56] (Fig. 5, at $M_e = 4.75$) are considerably lower than those of Young[57] for the flat plate, at $M_e = 4.53$. Part of this difference may be attributed to low readings for the Lee et al. data since the Kistler

Table 5 Skin friction for nonadiabatic wind-tunnel walls (Figs. 8 and 9)

Symbol	M_e	$Re_\theta\times10^{-3}$	T_w/T_{aw}	Ref.
×	4.7	24.5	0.54 → 0.75	56
○	7.4	36.5	0.32 → 0.49	27
◿	8.6	6.3	0.08 → 0.19	45
◐	4.2	13	1.0	55

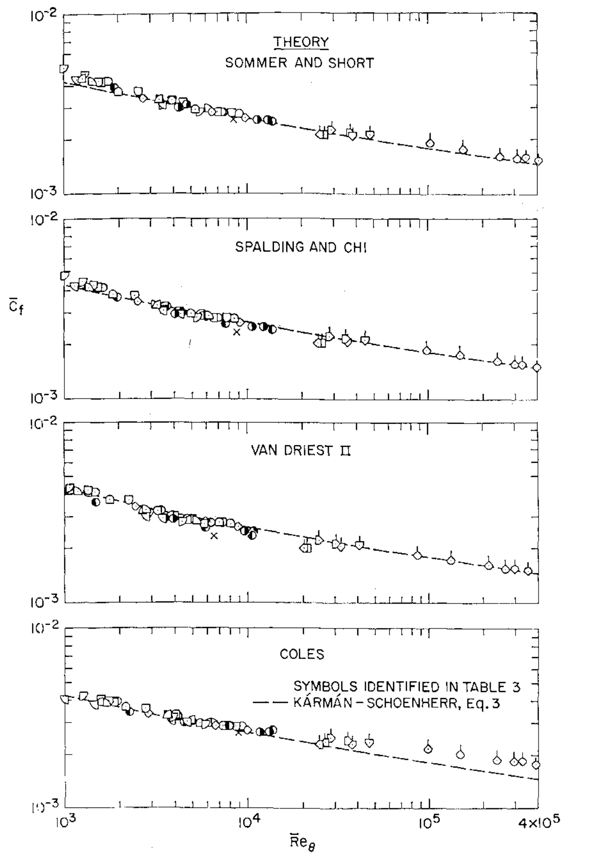

Fig. 4 Generalization of adiabatic-wall skin friction measured on wind-tunnel walls; C_f and Re_θ directly measured; $M_e = 1.8 \rightarrow 4.8$.

balance used by Hopkins and Keener,[41] when mounted side by side with the balance used by Lee et al.[56] in the NOL channel, gave skin-friction readings about 7% higher.

Nonadiabatic flat plates

Examination of the Hopkins and Keener data[41] in Fig. 6 indicates that any effect of boundary-layer trips on $\bar{C}_f(\bar{R}e_\theta)$ was within the experimental accuracy (compare open circles for trips with the filled circles). In addition, results from the injected model at angles of attack of 0° and 3° (flagged filled circles) generally agree with those for the stationary model (unflagged filled circles). For Neal's data[58] (Fig. 7, $T_w/T_{aw} = 0.57$) it appears that the method used herein to derive the origin of turbulent flow (continuous-line square symbol) gives better agreement with the other data than the method in which the maximum Stanton number was taken as the origin of turbulent flow (dashed-line square symbol). For all data presented at $T_w/T_{aw} > 0.3$, the theories of Van Driest II or Coles predict the skin friction to within about 10%, whereas the theories of Sommer and Short and Spalding and Chi underpredict the skin friction by 20 to 30%. At the lower temperature ratios, none of the theories predict the experimental variation of C_f with T_w/T_{aw}. It can also be observed that for the adiabatic wall points ($T_w/T_{aw} = 1$ in Fig. 7) there is a larger difference between experiment and theory for the Sommer and Short and Spalding and Chi theories than for the other two theories. Part of this result is believed related to the latter theories accounting for the effects of Mach number differently as already discussed for Fig. 3. The data of Wallace and McLaughlin[48] at $T_w/T_{aw} = 0.14$ are believed to be relatively lower than the other data primarily due to the low T_w/T_{aw} rather than the high Reynolds number as evidenced by examining Figs. 6 and 7.

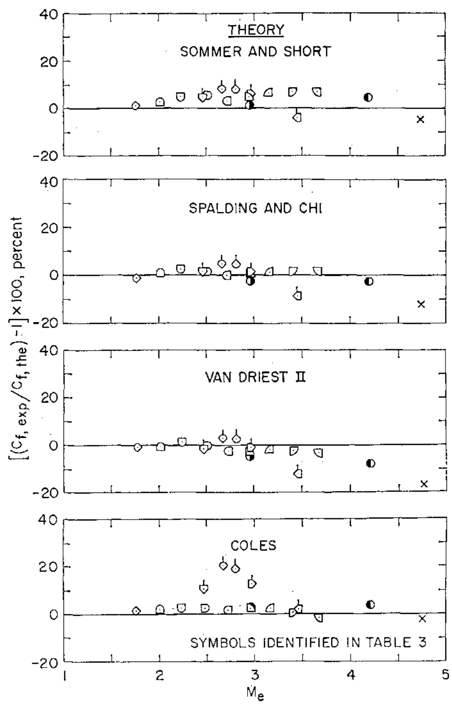

Fig. 5 Effect of Mach number on predictions of adiabatic-wall skin friction measured on wind-tunnel walls; C_f and Re_θ directly measured.

Nonadiabatic wind-tunnel walls

In the comparisons between theory and experiment for nonadiabatic walls, the theories of Van Driest II, Spalding and Chi, and Coles were used as originally developed and are based, therefore, on the Crocco linear variation of total temperature with velocity through the boundary layer. Although it is known that the total temperature distribution on wind-tunnel walls near the nozzle is generally closer to a quadratic than a linear distribution with velocity[27,45,46] over the outer part of the boundary layer, no accurate and detailed measurements have been made very close to the wall where the local skin friction is governed. Wallace[45] found that Re_θ from a quadratic temperature distribution as measured in his wall test was about four times larger than the Re_θ from an assumed Crocco linear temperature distribution and that the predicted skin friction as a result was halved.

Comparison of the results in Fig. 8 with those in Fig. 4 indicates that the correlation of the wind-tunnel wall data onto a single line by any of the theories is considerably inferior for the case with heat transfer. In Fig. 9 at $T_w/T_{aw} \simeq 0.4$, only the theory of Van Driest II surprisingly gives good predictions of skin friction. Again, the data of Lee et al.,[56] which are the only channel data presented, are generally low relative to the other results. Predictions from Coles' theory were evidently again affected by the high Reynolds numbers.

Admittedly, any agreement between skin-friction predictions based on assumed isobaric flow from the origin of turbulence with measured skin friction in boundary-layer flow known to be in dynamic nonequilibrium because of a flow expansion process in the nozzle must be viewed as inconclusive until this problem is studied further and additional experimental results are obtained.

Reynolds Analogy Factor

The earliest empirical correlation for Reynolds analogy factor was reported by Colburn,[60] who found for incompressible flow that a modified Reynolds analogy factor can be ex-

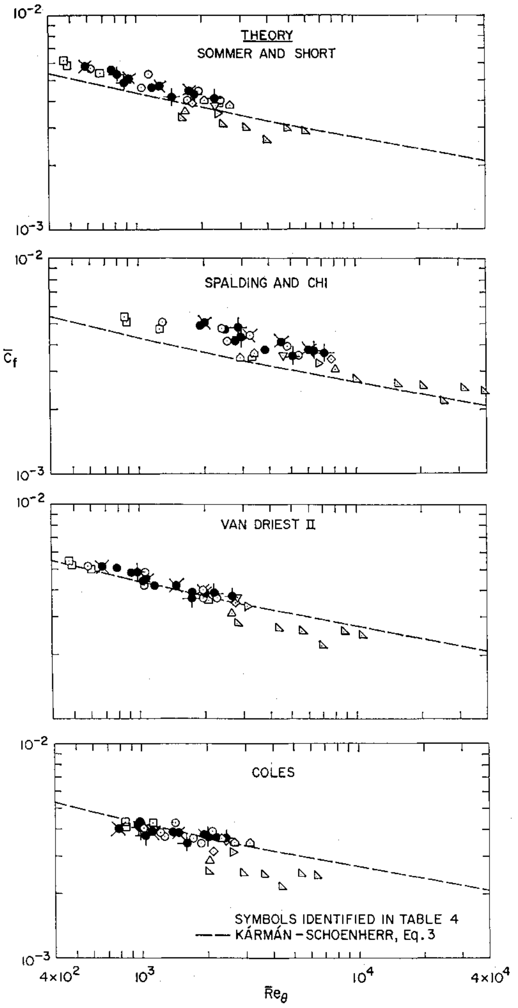

Fig. 6 Generalization of nonadiabatic-wall skin friction measured on flat plates; C_f directly measured; M_e = 2.8 → 7.4.

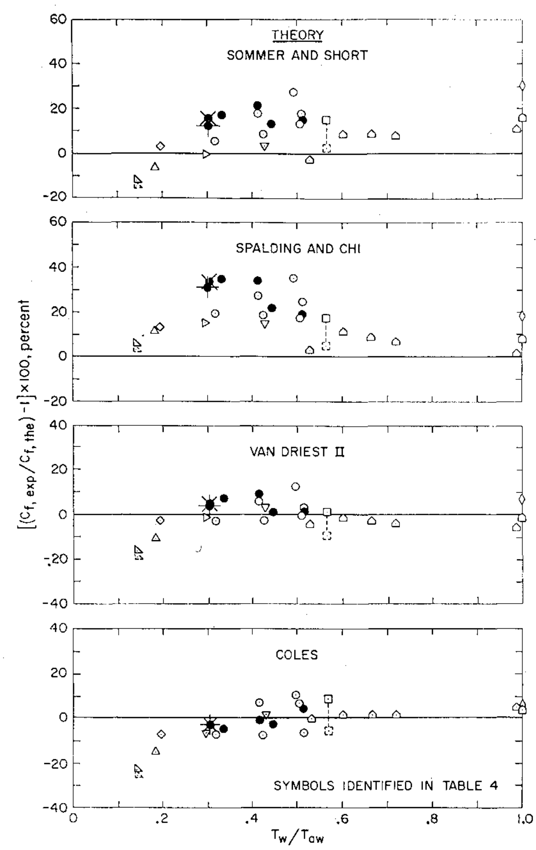

Fig. 7 Effect of wall-temperature ratio on predictions of skin friction measured on flat plates; C_f directly measured; M_e = 2.8 → 7.4.

pressed similarly to that for laminar flow as

$$2C_h/C_f = Pr^{-2/3} \tag{45}$$

For Pr = 0.72, this equation yields 1.25. More recent surveys of data at higher speeds including supersonic flow suggest slightly lower values. Seiff[61] recommends a value of 1.22, and Chi and Spalding[62] recommend a value of 1.16. Both values are subject to a mean deviation of the order of 8%. Figure 10, taken from Chi and Spalding,[62] illustrates the scatter in the Reynolds analogy factor, plotted as a function of Mach number for nearly adiabatic wall conditions. Chi and Spalding recommend that the value of 1.16 be used irrespective of Mach number, Reynolds number, and wall-to-adiabatic-wall temperature ratio.

At hypersonic Mach numbers and with nonadiabatic wall conditions, Hill,[12] Wallace,[45] and Perry and East[63] report Reynolds analogy factors of about unity, which is the value originally proposed by Reynolds.[42] The data in Ref. 56 also indicate a value of unity. This value is supported by recent experiments at Ames[27] in which local surface heat transfer and skin friction were measured simultaneously at the same axial location on a flat plate. It was found for M_e = 6.8 and 7.4 and T_w/T_{aw} = 0.3 that the heat flux-shear ratio was

$$\dot{q}_w U_e/\tau_w(H_w - H_e) = 0.84 \rightarrow 0.94 \tag{46}$$

These values are 10–20% lower than the value of 1.02 obtained by assuming $2C_h/C_f$ = 1.17 and r = 0.9. The data were not complete enough, however, to evaluate explicitly the Reynolds analogy and recovery factors. If a recovery factor of 0.9 is assumed, the corresponding value of the Reynolds analogy factor would be 1.0. A recent survey by Cary[65] shows that for Mach numbers greater than 5, the data are insufficient to determine empirically the dependence of the Reynolds analogy factor on Mach number, Reynolds number, and wall-to-adiabatic-wall temperature ratio. Whatever data are available have much scatter as shown in Fig. 11 taken from Cary's paper.[65]

In summary, the Reynolds analogy factor is known to an acceptable accuracy only for near adiabatic wall conditions at supersonic and lower Mach numbers. For these flow conditions, a constant value of 1.2 is recommended. With considerable wall cooling or at hypersonic speeds, the available data are insufficient to justify recommending unequivocally a Reynolds analogy factor, although a number of experiments[12,27,45,56,63] suggest a value of unity. It will be shown in the next section that the use of a Reynolds analogy factor of unity results in the best correlation between the heat-transfer and skin-friction data recently obtained at Ames Research Center.

Heat Transfer

The preceding discussion has indicated that for supersonic and lower speeds and near adiabatic wall conditions, the heat transfer can be predicted to a reasonable accuracy using a Reynolds analogy factor of 1.2 and any of the four skin-friction theories. The present discussion will be limited to recent heat-transfer data taken at hypersonic speeds and with surfaces cooled well below the adiabatic wall temperature. The data chosen for evaluation are restricted to those where the energy thickness can be calculated either from an integration of the energy flux across the boundary layer or an integration of the surface heat transfer beginning near the leading edge.

The heat-transfer data are compared with the four theories in Fig. 12 in a manner analogous to the previous presentation

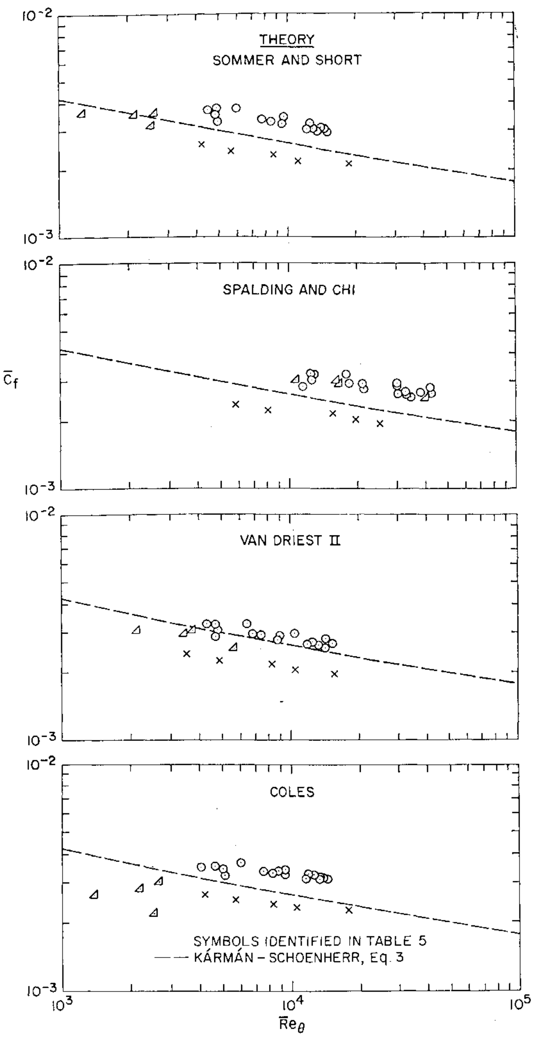

Fig. 8 Generalization of nonadiabatic-wall skin friction on wind-tunnel walls; C_f and Re_θ directly measured $M_e = 4.7 \rightarrow 8.6$; $T_w/T_{aw} = 0.08 \rightarrow 0.75$.

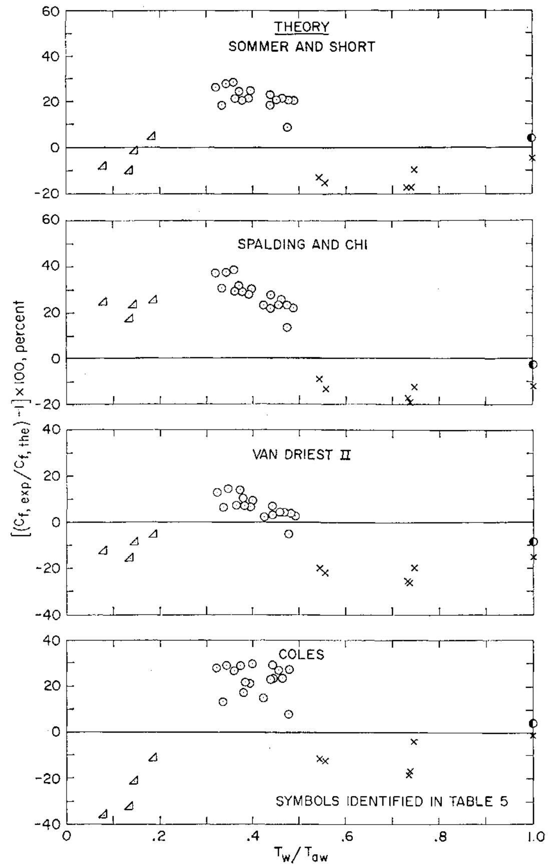

Fig. 9 Effect of wall-temperature ratio on predictions of skin friction measured on wind-tunnel walls; C_f and Re_θ directly measured; $M_e = 4.7 \rightarrow 8.6$.

of skin-friction data. The symbols are identified in Table 6 with references and flow conditions. The listed energy thickness Reynolds numbers are average values, each point plotted in Fig. 12 representing an average over the portion of the plate with fully developed turbulent flow. A recovery factor of 0.9 and a Reynolds analogy factor of 1.0 have been used.

The theories of Van Driest and Coles generally predict the heat transfer within 10% for $T_w/T_{aw} \gtrsim 0.3$. The theory of Sommer and Short tends to underpredict the data by 15%. The theory of Spalding and Chi generally underpredicts the heat-transfer data by 20% over the entire wall temperature range. None of the theories predicts the data for very low wall temperature ratios ($T_w/T_{aw} \lesssim 0.3$).

Further analyses also indicated that the theory of Spalding and Chi combined with a Reynolds analogy factor of 1.2 would predict heat-transfer rates comparable to the theory of Van Driest with an analogy factor of 1.0 for $T_w/T_{aw} > 0.3$. However, the latter approach is recommended since then the same theory would then be applicable to both skin-friction and heat-transfer predictions. A word of caution is in order

Table 6 Heat-transfer data (Fig. 12)

Symbol	Model	M_e	$Re_\Gamma \times 10^{-3}$	T_w/T_{aw}	Ref.
○	5° cone	6.6	4	0.1→0.3	27
□	15° cone	4.9	4	0.15→0.52	27
◇	5° cone	5.0	4	0.19→0.58	27
△	Flat plate	6.8,7.4	5	0.32	27
▽	Flat plate	6.0	5	0.22→0.76	28
⌂	Flat plate	6.8	3	0.54	58
⊿	Flat plate	5.1	3	0.64→0.83	9
◺	Hollow cylinder	6.0	3	0.44→0.50	64

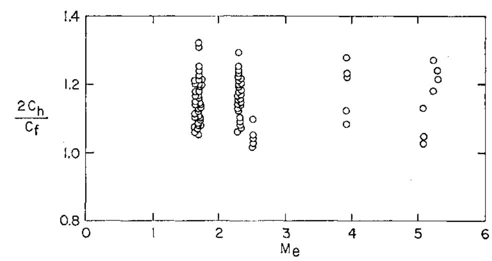

Fig. 10 Effect of Mach number on Reynolds analogy factor; $0.9 < T_w/T_{aw} < 1.1$ (Ref. 62).

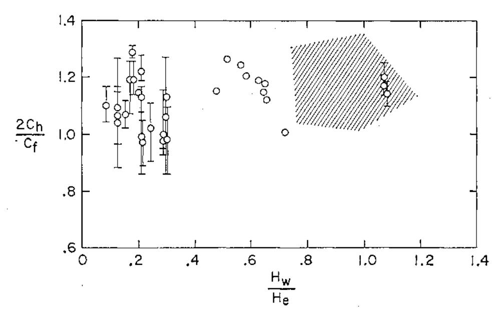

Fig. 11 Effect of wall-enthalpy ratio on Reynolds analogy factor; $1.5 < M_e < 11.7$ (Ref. 65).

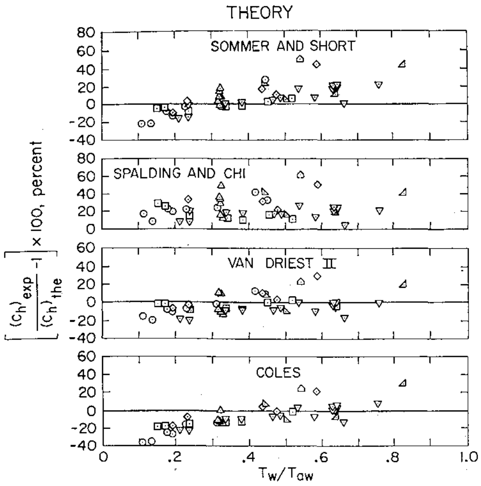

Fig. 12 Effect of wall-temperature ratio on predictions of heat transfer; $M_e = 4.9 \rightarrow 7.4$; $2C_h/C_f = 1.0$.

since there are several experimental points[9,27,58] that show high heat-transfer rates that can be predicted by using the Van Driest or Coles theory with a Reynolds analogy factor of 1.2.

Concluding Remarks

It is suggested that the Van Driest II theory be used to predict the turbulent skin friction for the design of supersonic and hypersonic vehicles until additional direct measurements of skin friction are made. This theory with an assumed Reynolds analogy factor of 1.0 should also be used to predict the heat transfer to surfaces exposed to hypersonic Mach numbers. For heat-transfer predictions to near adiabatic surfaces exposed to supersonic and lower Mach numbers, the use of a Reynolds analogy factor of 1.2 is recommended.

For the incompressible case, also required for compressible transformation theories such as that of Van Driest, additional direct measurements of skin friction are needed at extremely high and low Reynolds numbers before the Kármán-Schoenherr formula can be accepted without reservation.

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